Open Source, Dual Licensing and Software Competition

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Abstract

To distribute software, commercial firms have the opportunity to use some dual licensing strategy i.e. to provide their software under two different licensing terms (proprietary and open source). In this paper, we investigate the relevance and impacts of such distribution strategy in the presence of an already existing open source software. In this competitive setting, we determine in which conditions this strategy may be profitable for the commercial firm but also analyses the impact of such strategy on "traditional" open source communities and users.

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1 Introduction

We consider in this paper a situation where a commercial software editor competes with an Open Source Software (OSS) provider. To increase its sales/profits, we design a model in which the editor may introduce an OS version of its software through dual licensing. Doing so, he may capture part of OSS existing users. Conversely, this may also decrease its sales through the regular (proprietary) channel. We discuss this alternative distribution strategy through a theoretical modeling. We consider the effects of several types of software differentiation and distribution mode on competition and welfare in a framework characterized by the prevalence of both development and use externalities.

Section 2 presents the model and the benchmark case. Section 3 introduces the DL and characterize the possible equilibria with dual licensing. Section 3 analyzes the DL decision of the firm. Section 4 concludes.

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2 The Model

We consider the competition between platform software. A platform software is a software that developers use to build their own pieces of software. There are initially two platforms. One is released under a GPL license and is produced by an Open Source community of developers (further coined OS). The other (platform) software is produced by a commercial (i.e. profit motivated) editor.

2.1 The software editor

In the benchmark case, the commercial editor only distributes this software under a proprietary license terms (i.e. closed source code and not-free software, further coined P) and sells it at price \( p \) to form its revenues. With Dual Licensing (DL), it has the opportunity to distribute an additional version of its software under an open-source distribution. To keep things simple, let us assume that the original software is produced without cost\(^1\). It is reasonable to suppose that delivering the same software under different license terms does not imply specific costs\(^2\). Thus, using the DL does not generate additional costs. The editor maximizes its profit with respect to the license cost of the proprietary software and given the above simplifications, its profit \( \pi \) is simply defined by its revenues. Thus, \( \pi(p) = p m_p \) (where \( m_p \) denotes the number of adopters of the proprietary software).

2.2 User-Developers

There exists \( m \) potential software users. Without loss of generality, the total number of users is normalized to 1. Users have specific and different needs and any type software cannot fill any of these needs. Such heterogeneity requires for developers to create additional pieces of code so as to adapt the original software to its needs.

To depict this type of heterogeneity, let us assume that users are uniformly distributed on a unitary segment. The additional pieces of code require a development effort that we can depict through an additional cost which increases with the distance to the location of the software on the segment. By convention, let us assume that the proprietary software is located at location 0 on the unitary segment while the OS software is located at location 1, opposite to the proprietary software\(^3\).

We can thus simply write the utility of the proprietary software derived by the developer \( i \) when using the proprietary software \( (U_p) \) by \( U_p(i) = v - \alpha i - p \) where \( v \), \( \alpha i \) and \( p \) depict respectively the benefits derived from the P software, the additional development cost \( (\alpha > 0) \) associated with this software, function of the location of user \( i \), and the fees paid to the firm by the proprietary software users.

Similarly to the P software, users also need to write additional lines of code and develop new pieces of software when they adopt the OS software. The utility derived from OS adoption \( (U_{os}) \) is then \( U_{os}(i) = u - \beta (1 - i) \) where \( u \) depicts the benefits derived

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\(^1\)Since software production costs are essentially fixed costs, the introduction of such costs would not qualitatively alter the results.

\(^2\)The only cost would be the distribution cost that is negligible.

\(^3\)OS projects historically developed in reaction to proprietary standards because closed-source software were targeted to some special needs and could not able all developers’ needs. The two platforms are then located opposite to each other so as to reflect this complementarity.
from the OS software and where $\beta(1 - i)$ is the additional development cost ($\beta > 0$). If we consider that the OS community is able to provide a larger development support, we can also suppose that $\alpha > \beta$.

2.3 Outcomes in the Benchmark Case

The firm plays first and selects a licensing and price strategy. Observing software licensing terms and conditions, potential users play second and choose to adopt (or not) one among the different available software. The game is solved by backward induction. Since we are interesting in initially competitive outcomes, we focus on situations where the two software (OS, P) are active before the firm can choose to dual license. Depending on parameters, two situations can occur in the benchmark case.

In the first situation, all users initially adopt one software so that users divide among the P and the OS platform (see Appendix 1 for the computation of this equilibrium). This equilibrium is depicted by Lemma 1.

**Lemma 1 (P-OS). Initial Full Adoption.** If $u < v + \beta$ and if i) $v \leq 2\alpha + \beta$ and $u + v\beta/(2\alpha + \beta) > \beta$ or ii) $u < v + \beta$ and $v > 2\alpha + \beta$ and $u + 2\alpha + \beta > v$, then all user-developers adopt one of the two platforms. The commercial platform is sold at price $p^* = (v + \beta - u)/2$, the optimal profit of the commercial editor is equal to $\pi^* = (v + \beta - u)^2/(4(\alpha + \beta))$ and his share of market $i^* = (v + \beta - u)/(2(\alpha + \beta))$.

The restrictions on parameters depict the limitations imposed by (i) the coexistence of the two technologies, (ii) their attractiveness for potential users which are never interested by the reservation strategy and (iii) the profitability of this situation for the editor. The level of the fees, of the profit and the editor market share reveal, in concordance with intuition, that the position of the editor improves with the performance of the proprietary software and the disadvantages of the OS platform (improves with $v$, depreciates with $\beta$) and depreciates with each increase of the position of OS platform and with the decrease of its own position (increase of $v$ and $\alpha$).

In the second situation, some users initially adopt one software (P or OS) but also some does not adopt any of the two software since they do not obtain a positive utility with one of the two software (see Appendix 1 for the computation of this equilibrium). This equilibrium is depicted by Lemma 2.

**Lemma 2 (P-⊘-OS). Initial Partial Adoption.** If $v < 2\alpha$ and $\beta > 2u\alpha/(2\alpha - v)$, then a fraction $m_p^*_p = v/2\alpha$ of all potential users adopts the proprietary software, a fraction $m_{os} = u/\beta$ and the remaining part does not adopt any software. The commercial platform is sold at price $p^* = v/2$ and the optimal profit of the firm is then $\pi^* = v^2/4\alpha$.

The intuition for these conditions are as follows. For both software, the development costs (respectively measured by $\alpha$ and $\beta$) has to be sufficiently high compared to the utility of the two software (respectively $u$ and $v$). Otherwise, whenever these two costs are low relatively, all users would get a positive utility by adopting one of the two software.

When considering the two sets of parameters defining the two outcomes, we can check that these sets are incompatible. This rules out multiple equilibria in the benchmark case.
3 Equilibria with DL

The firm may introduce dual licensing if it is profitable to do so. Let us coin this hybrid platform as OSP (as it mixes some characteristics of both OS and P software). The introduction of the OSP software has several effects. First, the adopters of the OSP software may benefit from the functionalities \( v \) developed for the proprietary software without paying the license cost attached to the latter. Similarly to the other software, they incur an additional development cost depending on their location on the segment \( \alpha i \). Second, associated with the distribution of the OSP, some users developing on the OSP software will create new lines of codes of software. Unlike under the proprietary license terms, these lines of codes will inherit from the properties of the OS project and the firm will benefit from these to improve its proprietary software. Thus, this generates a spillover from the OSP to the P platform. Third, the OS and OSP may benefit from each other through a decrease in the development cost required when adopting these platforms. Indeed, part of the development effort of OS users may be used by OSP users to develop more efficiently and conversely. Yet, the magnitude of this last effect crucially depends on the compatibility degree between the two platforms. The more the two platforms are compatible, the easier will be the exchange of ideas and code across the two platforms. On the opposite side, if the two platforms are completely incompatible, no exchange across platforms will be possible. Let us capture the compatibility degree by parameter \( \lambda \) (with \( 0 < \lambda < 1 \)).

Introducing these three elements and supposing \( a > 0 \), we can reformulate the three utilities as follows:

\[
\begin{align*}
U_p(i) &= v - \alpha i + am_{osp} \\
U_{osp}(i) &= v - \alpha (1 - \lambda)i \\
U_{os}(i) &= u - \beta (1 - \lambda)(i - 1)
\end{align*}
\]

The introduction of OS translates into new potential situations. Again, we concentrate on some parameter configurations where OS and P are both active in the benchmark case (without DL). Several situations are now possible:

**Proposition 1** (P-OSP-OS). There exist admissible values of parameters compatible with an equilibrium such that an equilibrium situation exists in which some users adopt the OS platform, some adopt the OSP platform and the remaining users adopt the OS platform. In that case, the distribution between users is as follows: \( m_p^* = a(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1)(a + \alpha \lambda) \); \( m_{osp}^* = (v - u + \alpha(\lambda - 1))/2(\alpha + \beta)(\lambda - 1) \) and \( m_{os}^* = (a + 2\alpha \lambda)(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1)(a + \alpha \lambda) \) and the optimal price for the P platform is \( p^* = a(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1) \) (proof: see Appendix).

This depicts an equilibrium with full adoption (all user adopt exactly one platform).

**Proposition 2** (P-OSP-⊘-OS). When \( v < \alpha (1 - \lambda) \) and \( \beta > \frac{m_p^*}{v + \alpha(\lambda - 1)} \), an equilibrium situation exists when some users adopt the OS platform, some adopt the OSP platform,
some adopt the OS platform while some users do not adopt any platform. In that case, the distribution between users is as follows: \( m_p^* = \frac{av}{2\alpha(\lambda - 1)(a + \alpha \lambda)} \) ; \( m_{osp}^* = \frac{v(a + 2\alpha \lambda)}{2[\alpha \lambda (\lambda - 1)(a + \alpha \lambda)]} \) and \( m_{os}^* = \frac{u}{\beta(1 - \lambda)} \) and the optimal price for the P platform is \( p^* = \frac{av}{2\alpha(\lambda - 1)} \) (proof : see Appendix).

This depicts an equilibrium with partial adoption (some user adopt neither software).

**Proposition 3 (P-OSP).** If \( v < \alpha(1 - \lambda) \) and \( \beta > \frac{uv}{v + \alpha(\lambda - 1)} \), an equilibrium situation exists when some users adopt the P platform \( m_p^* = a/2(a + \alpha \lambda) \) and some adopt the OSP platform \( m_{osp}^* = 1 - m_p^* = 1 - a/2(a + \alpha \lambda) \). In that case, the introduction of the OSP platform induces the exit of the OS platform \( m_{os}^* = 0 \). The P software is sold at price \( p^* = a/2 \) and the profit of the firm is \( \pi^* = a^2/4(a + \alpha \lambda) \) (proof : see Appendix).

This shows how the introduction of the OSP platform can serve as a means to crowd out the OS platform. When \( \beta \) is sufficiently large, the adoption of the OS platform is no longer efficient for users that switch to the OSP platform. It is thus possible to switch from a P-OS or from a P-OSP equilibrium to a P-OSP equilibrium. Interestingly, the price charged by the firm is here only dependant on the size on the externality coming from the OSP. Everything else equal, when spillovers are large, the firm can charge a higher price to its direct customers (users of the P platform). In particular, the price does not depend on \( v \) since the amount of utility \( v \) is common to the two software. A practical implication of that is that the main driver of the price of the P-software is not the intrinsique value of the software but the amount of externality generated by the OSP software.

When considering the two sets of parameters defining the three outcomes, we can check that these sets are all mutually incompatible. This rules out multiple equilibria in the case with hybrid software.

### 4 DL decision

We here analyze the decision of the firm towards DL and its effect on market structure. Considering the potential equilibria of the game (in the benchmark case and with DL), we need to consider all the potential switches raised by the introduction of the DL licence. By comparing the different equilibrium conditions and check whether these are compatible, we can demonstrate that some switchers are not possible as revealed by the following table:

Clearly, this means that a situation where all users adopt one software (full adoption) without DL cannot lead to as situation with partial adoption when the DL is introduced. All other possible switch are possible.
4.1 P-OSP case

Let us first consider that the outcome with DL is of type P-OSP. Let us assume that $\lambda = 0$

4.1.1 Initial Full Adoption

Let us first consider the case of initial full adoption (i.e. P-OS equilibrium), the firm has an incentive to dual licence its software only when

$$\pi^*_P - OSP > \pi^*_P - OS.$$ 

This is the case when

$$\left( \begin{array}{c}
\beta < \sqrt{2} \alpha & \alpha + \beta < 2 \alpha + \beta \alpha < \frac{v^2}{2 \alpha + \beta} < u < v - \alpha \\
\alpha + \beta < 2 \alpha + \beta \alpha < \frac{v^2}{2 \alpha + \beta} < u < v - \alpha \alpha > \frac{(-u + v + \beta)^2}{(\alpha + \beta)}
\end{array} \right) \right.$$

$$\left( \begin{array}{c}
\sqrt{2} \alpha < \beta < \alpha + \sqrt{6} \alpha \alpha > \frac{(-u + v + \beta)^2}{(\alpha + \beta)}
\end{array} \right) \right.$$

$$\left( \begin{array}{c}
v > 2 \alpha + \beta \alpha < 2 \alpha + \beta \alpha < \frac{v^2}{2 \alpha + \beta} < u < v - \alpha \alpha > \frac{(-u + v + \beta)^2}{(\alpha + \beta)}
\end{array} \right) \right.$$

$$\left( \begin{array}{c}
\sqrt{6} \beta > 5 \alpha + \beta \alpha > \frac{(-u + v + \beta)^2}{(\alpha + \beta)}
\end{array} \right) \right.$$

$$\left( \begin{array}{c}
v > 2 \alpha + \beta \alpha < 2 \alpha + \beta \alpha < \frac{v^2}{2 \alpha + \beta} < u < v - \alpha \alpha > \frac{(-u + v + \beta)^2}{(\alpha + \beta)}
\end{array} \right) \right.$$

This condition is more restrictive than the condition of existence for this equilibrium ($v > \alpha, \alpha > 0$, and $u + \alpha < v$). This means that when the market is initially covered, implementing the DL strategy in that situation is not always profitable for the firm. As a corollary, excluding the OS platform through the introduction of DL is not always an optimal strategy and an 'accommodating strategy' should be used instead.

Proposition 4. When the market is initially fully covered and when it is profitable to DL, the marketshare of the P software with DL is always less than that without DL, and the price charged is higher.

This is the case when:

$$u + \alpha < v \alpha > u + v + \beta \alpha \left( \begin{array}{c}
v < 2 \alpha + \beta \alpha v > 2 \alpha + \beta \alpha > \frac{v^2}{2 \alpha + \beta} > \beta
\end{array} \right) \right.$$
\[(v > 2\alpha + \beta \& \& u + 2\alpha + \beta > v) \& \& ((\sqrt{2\alpha} > \beta \& \& \beta > 0) \land \sqrt{6\beta} > 5\alpha + \beta)\]

This proposition has some practical implications. It implies that the OSP acts as a substitute for both OS and P. In other terms, when it is profitable to DL, the market share of the P platform should always decrease. Despite the loss in marketshare, DL is profitable thanks to the spillover coming from the OSP. In turn, these spillovers allow the firm to better price its product even if the number of regular customers is reduced.

**Remark 1.** The comparison between the P-OS and the P-OSP is ambiguous from a welfare perspective.

Total welfare (including the profit of the firm) is increasing when ...

\[
\alpha > 0 \& \& \left(\frac{(-u + v + \beta)^2 + u(\alpha + \beta)}{\alpha + \beta}\right) > 0 \& \& ((u + \alpha < v) \& \& \left(\begin{array}{c}
2v > 7\alpha + 6\beta \\
2v < 7\alpha + 6\beta \& \& u + 2\alpha + \beta > v \& \& \left(\begin{array}{c}
7v + 5\beta + 2\sqrt{4v^2 + 4\alpha^2 + \beta} - 2\beta^2 - 2v(7\alpha + 5\beta) \leq 11u + 4\alpha \& \& \\
7v + 5\beta + 2\sqrt{4v^2 + 4\alpha^2 + \beta} - 2\beta^2 - 2v(7\alpha + 5\beta) > 4v \land \& \& \\
(\sqrt{6\beta} > 5\alpha + \beta) \land (\sqrt{2\alpha} > \beta \& \& \beta > 0) \land (\sqrt{2\alpha} < \beta \& \& \beta + \beta > 5\alpha + \beta) \end{array}\right)\end{array}\right)\left(\begin{array}{c}
2v \geq 7\alpha + 6\beta \& \& \\
(2v < 7\alpha + 6\beta \& \& 7v + 5\beta + 5\beta > 4v) \land \& \& \\
(\sqrt{6\beta} > 5\alpha + \beta) \land (\sqrt{2\alpha} > \beta \& \& \beta > 0) \land (\sqrt{2\alpha} < \beta \& \& \beta + \beta > 5\alpha + \beta) \end{array}\right)\right)\right)\]

Welfare of P-users is increasing when (same conditions as previously) ...

\[
\beta > 0 \& \& \left(\begin{array}{c}
\beta + \frac{u\alpha}{\beta + 2\beta} > u \& \& \\
4v \geq 11\alpha + 8\beta \land (3\alpha + 2\beta + \sqrt{6\alpha^2 + 10\alpha\beta + 4\beta^2} < 2v) \land \& \& \\
(\sqrt{6\beta} > 5\alpha + \beta) \land (\sqrt{2\alpha} > \beta \& \& \beta > 0) \land (\sqrt{2\alpha} < \beta \& \& \beta + \beta > 5\alpha + \beta) \end{array}\right)\right)\]

\[
\left(\begin{array}{c}
(\alpha + \beta)^2(4v - 12\alpha + 3\alpha^2 - 8\alpha^3 + 2\alpha^3) < (3\alpha + 2\beta) \land \& \& \\
(3\alpha + 2\beta) + \sqrt{6\alpha^2 + 10\alpha\beta + 4\beta^2} = 2v \land \& \& \\
(\sqrt{6\beta} > 5\alpha + \beta) \land (\sqrt{2\alpha} > \beta \& \& \beta > 0) \land (\sqrt{2\alpha} < \beta \& \& \beta + \beta > 5\alpha + \beta) \end{array}\right)\right)\]

\[
\left(\begin{array}{c}
\beta > \frac{u\alpha}{\beta + 2\beta} \land (3\alpha + 2\beta + \sqrt{6\alpha^2 + 10\alpha\beta + 4\beta^2} = 2v) \land \& \& \\
(\sqrt{6\beta} > 5\alpha + \beta) \land (\sqrt{2\alpha} > \beta \& \& \beta > 0) \land (\sqrt{2\alpha} < \beta \& \& \beta + \beta > 5\alpha + \beta) \end{array}\right)\right)\]

\[
\left(\begin{array}{c}
(\alpha + \beta)^2(4v - 12\alpha + 3\alpha^2 - 8\alpha^3 + 2\alpha^3) < (3\alpha + 2\beta) \land \& \& \\
(3\alpha + 2\beta) + \sqrt{6\alpha^2 + 10\alpha\beta + 4\beta^2} = 2v \land \& \& \\
(\sqrt{6\beta} > 5\alpha + \beta) \land (\sqrt{2\alpha} > \beta \& \& \beta > 0) \land (\sqrt{2\alpha} < \beta \& \& \beta + \beta > 5\alpha + \beta) \end{array}\right)\right)\]

\[
\left(\begin{array}{c}
\beta > \frac{u\alpha}{\beta + 2\beta} \land (3\alpha + 2\beta + \sqrt{6\alpha^2 + 10\alpha\beta + 4\beta^2} = 2v) \land \& \& \\
(\sqrt{6\beta} > 5\alpha + \beta) \land (\sqrt{2\alpha} > \beta \& \& \beta > 0) \land (\sqrt{2\alpha} < \beta \& \& \beta + \beta > 5\alpha + \beta) \end{array}\right)\right)\]
Welfare all users is increasing when ...

\[ \alpha > 0 \land \& \& \left( \frac{\sqrt{v^{2}+\beta^{2}}}{\lambda} v^{2}+\alpha^{2} > 0 \land \& \& \left( u + \alpha < v \land \& \& \left( v > 2a + \beta \land \& \& u + 2a > v \land \& \& v + \frac{\sqrt{v^{2}+\beta^{2}}}{\lambda} v^{2} > 2a \right) \right) \right) \]

\[ \left( 7v + 5\beta + 2\sqrt{v^{2}+\alpha^{2}} > 0 \land \& \& \left( 7v + 5\beta + 2\sqrt{v^{2}+\alpha^{2}} \leq 11u + 4\alpha \land \& \& \left( 2v < 7a + 6\lambda \land \& \& 7a + 5\beta + \sqrt{v^{2}+\beta^{2}} > 4v \land \& \& 9v^{2}+\beta^{2} + 2\alpha^{2} > 2a^{2} \right) \right) \right) \]

4.1.2 Initial Partial Adoption

Let us now consider the case P- OS (partial adoption) as the initial case. It is profitable for the firm to implement a DL strategy when \( \pi_{P-\os}^{*} > \pi_{P-\os}^{*} \). This translates into the following condition (for any \( \lambda \)):

\[ \left( v > 0 \land \& \& \sqrt{v^{2}+\alpha^{2}} > a + v \land \& \& \left( \frac{2a^{2} v^{2}}{\lambda} + \beta > 0 \land \& \& \left( a v + \alpha \lambda < 1 \land \& \& \left( a \neq 0 \& \& \left( \alpha > 0 \land \& \& \left( \frac{a^{2}}{\lambda} > \frac{v^{2}}{\lambda} + \alpha \land \& \& \left( 2a^{2} v^{2} > 4v^{2} + \beta > 0 \land \& \& \left( a v + \alpha < 0 \land \& \& \left( a > 0 \land \& \& \left( \frac{a^{2}}{\lambda} > \frac{v^{2}}{\lambda} + \alpha \land \& \& \left( \frac{a^{2}}{\lambda} > \frac{v^{2}}{\lambda} + \alpha \right) \right) \right) \right) \right) \right) \right) \right) \right) \]
5 Conclusion

References


Appendix

Appendix 1: Computation of the benchmark case

Equilibrium of type P/OS

The user (noted \(i_{p/OS}\)) indifferent between the two software is defined by \(U_p(i_{p/OS}) = U_{os}(i_{p/OS})\). Hence, \(i_{p/OS} = (v + \beta - u - p)/(\alpha + \beta)\). From that, we deduce the expression of the profit \(\pi = p i_{p/OS}\) and obtain from the FOC that \(p^* = (v + \beta - u)/2\) and \(m^* = (v + \beta - u)/(2(\alpha + \beta))\). The optimal profit of the firm is then \(\pi^* = (v + \beta - u)^2/(4(\alpha + \beta))\). Putting together the boundary condition for \(i_{p/OS}(0 < i_{p/OS} < 1)\), the condition for the utility of all users to be strictly positive \((U_p(i_{p/OS}) > 0)\) and the second-order condition, we obtain the following existing conditions for this equilibrium: \(u < v + \beta\) and \(v < 2\alpha + \beta\) and \(u + v\beta/(2\alpha + 2\beta - 3\alpha\beta - 2\beta^2) > \beta\) or if \(u < v + \beta\) and \(v > 2\alpha + \beta\) and \(u + 2\alpha + \beta > v\). From these conditions, we can deduce the expression of P users’ surplus \(W_p = \int_0^{i_{p/OS}} U_p(i)di = (v+\beta-u)(u(4\alpha+5\beta)-\beta(v+\beta))/(8(\alpha+\beta)^2)\), of OS users’ surplus \(W_{os} = \int_{i_{p/OS}}^1 U_{os}(i)di = (v+\beta-u)(\alpha\alpha+2\alpha\beta+2\alpha\beta-3\alpha\beta-2\beta^2)/(8(\alpha+\beta)^2)\), the total surplus of all users \(W_{users} = W_p + W_{os} = (v+\beta-u)(7\alpha+6\beta)/(8(\alpha+\beta)^2)\) and the total welfare \(W\) including the profit of the firm \(W = W_{users} + \pi^* = (v+\beta-u)(5\alpha+3\beta-\beta)/(8(\alpha+\beta)^2)\).

Equilibrium of type P/\(\bigcirc\)/OS

In the second situation, the users located close to 0 adopt the P software first, those located close to 1 adopt the OS software first.

Between these two populations, some users obtain a negative utility with the two software and do not adopt neither software. The user (noted \(i_{p/\bigcirc}\)) is defined by a null utility when adopting the P software \(U_p(i_{p/\bigcirc}) = 0\). By definition of this equilibrium, we impose that \(U_{os}(i_{p/\bigcirc}) < 0\) (if not, we would be in the previous situation with full adoption). Similarly, the user (noted \(i_{\bigcirc/OS}\)) is defined by a null utility when adopting the OS software \(U_{os}(i_{\bigcirc/OS}) = 0\). Again, by definition of this equilibrium, we impose that \(U_p(i_{\bigcirc/OS}) < 0\) (if not, we would be in the previous situation with full adoption). These two restrictions imply that \(0 < i_{p/\bigcirc} < i_{\bigcirc/OS} < 1\) (boundary conditions).
We deduce that \( i_{p/\circ} = (v - p/\alpha) \) and \( i_{\circ/os} = (1 - u)/\beta \). From that, we deduce the expression of the profit \( \pi = p_i p/\circ \) and obtain from the FOC that \( p^* = v/2 \) and \( m_{p^*} = v/2\alpha \). The optimal profit of the firm is then \( \pi^* = v^2/4(\alpha) \). Putting together the boundary conditions and the second-order condition, we obtain the following existing conditions for this equilibrium: \( v < 2\alpha \) and \( \beta > 2ua/(2\alpha - v) \). In these conditions, we can deduce the expression of P users’ surplus \( (W_p = \int_0^{i_{p/\circ}} U_p(i) di = v^2/8\alpha) \), of OS users’ surplus \( (W_{os} = \int_{i_{\circ/os}}^{i_{p/\circ}} U_{os}(i) di = 2u - 3u^2/2\beta - \beta/2) \), the total surplus of all users \( (W_{users} = W_p + W_{os} = (16u + v^2/\alpha - 12u^2/\beta - 4\beta)/8) \) and the total welfare \( W \) including the profit of the firm \( (W = 2u + 3u^2/8\alpha - 3u^2/2\beta - \beta/2) \).

Considering the The conditions for the first type of equilibrium to occur are incompatible with those for the second type of equilibrium to occur so that we can rule out multiple equilibria.

**Note:** The existing conditions for equilibria of type P-OSP-OS, P-OSP-\( \circ \)-OS and P-OSP are all mutually exclusive. This means that starting with one parameter set, only one type of equilibrium can occur.

**Computation of the DL case**

**The P-OSP-OS case**

Let us first consider the case where all users adopt. By increasing values of \( i \), they adopt the P, the OSP and then the OS software. The user (noted \( i_{p/\circ} \)) indifferent between the two software is defined by \( U_p(i_{p/\circ}) = U_{OSP}(i_{p/\circ}) \). Hence, \( i_{p/\circ} = (am_{\circ/os} - p)/(\alpha \lambda) \). Similarly, the user indifferent between the OSP and the OS software (noted \( i_{\circ/os} \)) is defined by \( U_{OSP}(i_{\circ/os}) = U_{os}(i_{\circ/os}) \). Thus, we obtain, \( i_{\circ/os} = (u - v + \beta(\lambda - 1))/((\alpha + \beta)(\lambda - 1)) \). This case is valid only when \( 0 < i_{p/\circ} < i_{\circ/os} < 1 \) (boundary conditions). From this expression, we compute the expression of the profit \( \pi = p_i p/\circ \) and obtain from the FOC that \( p^* = a(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1) \) and \( m_{p^*} = a(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1)(a + \alpha \lambda) \). We also deduce the number of OSP and OS users at equilibrium: \( m_{\circ/os} = (v - u + \alpha(\lambda - 1))/(\alpha + \beta)(\lambda - 1) \) and \( m_{os} = (a + 2a\lambda)(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1)(a + \alpha \lambda) \). From that, we can compute the profit of the firm and the surplus of all categories of users. The optimal profit of the firm is then \( \pi^* = a^2(v - u + \beta(1 - \lambda))^2/(4(\alpha + \beta)^2(\lambda - 1)^2(a + \alpha \lambda)) \). Putting together the boundary conditions and the second-order condition, we obtain the following existing conditions for this equilibrium: XXX. In these conditions, we can deduce the expression of P users’ surplus \( W_p \), of OSP users’ surplus \( W_{osp} \) of OS users’ surplus \( W_{os} \), the total surplus of all users \( W_{users} \) (with \( W_{users} = W_p + W_{osp} + W_{os} \)):

\[
W_p = \int_0^{i_{p/\circ}} U_p(i) di = \ldots \\
W_{osp} = \int_{i_{p/\circ}}^{i_{\circ/os}} U_{osp}(i) di = \ldots \\
W_{os} = \int_{i_{\circ/os}}^{1} U_{os}(i) di = \ldots \\
W_{users} = \ldots
\]

Putting together the different conditions of existence of such equilibrium (boundary conditions, positive profit, positive second-order derivative, positive level of utility for all
Figure 2: Equilibrium values (profit, surplus)

Profit optimal : 
\[
\frac{\alpha (u - \beta + \beta \lambda)}{2 (\alpha + \beta) (-1 + \lambda)}
\]

Profit optimal (P) :
\[
\frac{\alpha (u - \beta + \beta \lambda)}{2 (\alpha + \beta) (-1 + \lambda) (\alpha + \beta \lambda)}
\]

Profit optimal (OSP) :
\[
\frac{(a + 2 \lambda) (u - \beta + \beta \lambda)}{2 (\alpha + \beta) (-1 + \lambda) (\alpha + \beta \lambda)}
\]

Profit optimal (OS) :
\[
\frac{-u + \gamma - (1 + \lambda)}{\alpha + \beta (-1 + \lambda)}
\]

Profit optimal :
\[
\frac{\sigma^2 (u + \gamma + \beta - \beta \lambda)}{4 (\alpha + \beta)^2 (-1 + \lambda)^2 (\alpha + \beta \lambda)}
\]

Calculate welfare

Surplus utilisateurs P :
\[
\alpha (u - \beta + \beta \lambda) (4 \gamma + (\alpha + \beta) (-1 + \lambda)) + (\gamma (\alpha + \beta) + \alpha \beta (1 + \lambda) (-1 + \lambda) + u (4 \beta (1 + \lambda) + \alpha (-3 + 2 \lambda)))
\]

\[
\frac{\theta (\alpha + \beta)^2 (-1 + \lambda)^2 (\alpha + \beta \lambda)}
\]

Surplus utilisateurs OSP :
\[
\frac{(a + 2 \lambda) (u - \beta + \beta \lambda)}{2 (\alpha + \beta) (-1 + \lambda) (\alpha + \beta \lambda)}
\]

\[
\theta (\alpha + \beta)^2 (-1 + \lambda)^2 (\alpha + \beta \lambda)
\]

General spellings:
Possible spelling error: new symbol name ’surplusmax’ is similar to existing symbol ’surplusmax’. Plus,

Surplus utilisateurs OS :
\[
\frac{\theta (\alpha + \beta)^2 (-1 + \lambda)^2}{2 (\alpha + \beta) (-1 + \lambda)}
\]

Surplus total utilisateurs :
\[
\frac{1}{2} \left( \frac{1}{(\alpha + \beta)^2 (-1 + \lambda)^2} \right) \left[ \frac{\theta (\alpha + \beta)^2 (-1 + \lambda)^2 (\alpha + \beta) + \alpha \beta (1 + \lambda) (-1 + \lambda) + u (4 \beta (1 + \lambda) + \alpha (-3 + 2 \lambda))}{(\alpha + \beta \lambda)^2} \right]
\]

Surplus total utilisateurs + filtre :
\[
\frac{1}{2} \left( \frac{1}{(\alpha + \beta)^2 (-1 + \lambda)^2} \right) \left[ \frac{\theta (\alpha + \beta)^2 (-1 + \lambda)^2 (\alpha + \beta) + \alpha \beta (1 + \lambda) (-1 + \lambda) + u (4 \beta (1 + \lambda) + \alpha (-3 + 2 \lambda))}{(\alpha + \beta \lambda)^2} \right]
\]
users), we obtain the following conditions for such equilibrium to occur.

\[
\begin{align*}
0 &< \beta - \frac{u - v}{1 - \lambda} < \lambda < 1, \\
&< \beta - \frac{u - v}{1 - \lambda} < \lambda < 1, \\
&< \beta - \frac{u - v}{1 - \lambda} < \lambda < 1, \\
&< \beta - \frac{u - v}{1 - \lambda} < \lambda < 1.
\end{align*}
\]

Figure 3: Conditions for an P-OSP-P equilibrium (no restriction on parameters)

Let us then consider the case with DL but where some users do not adopt. Considering increasing values of \(i\), users adopt the P platform, then the OSP platform. Those users with the highest values of \(i\) adopt the OS software. Finally, those users characterized by "intermediate" values of \(i\) do not adopt any software. This characterizes four critical values of \(i\). The user (noted \(i_{p/osp}\)) indifferent between the two software is defined by

\[
U_{p} (i_{p/osp}) = U_{osp} (i_{p/osp}).
\]

Hence,

\[
i_{p/osp} = \frac{a_{m} - p}{v}(1 + \lambda).
\]

Figure 4: Conditions for an P-OSP-OS equilibrium with \(\alpha > \beta\)

The P-OSP-⊘-OS case

Let us then consider the case with DL but where some users do not adopt. Considering increasing values of \(i\), users adopt the P platform, then the OSP platform. Those users with the highest values of \(i\) adopt the OS software. Finally, those users characterized by "intermediate" values of \(i\) do not adopt any software. This characterizes four critical values of \(i\). The user (noted \(i_{p/osp}\)) indifferent between the two software is defined by

\[
U_{p} (i_{p/osp}) = U_{osp} (i_{p/osp}).
\]

Hence,

\[
i_{p/osp} = \frac{a_{m} - p}{v}(1 + \lambda).
\]

For values of \(i\) higher than \(i_{p/osp}\), users adopt the OSP platform as long as

\[
U_{osp} (i) > 0.
\]

Considering increasing values of \(i\), the 'last' user to adopt OSP (noted \(i_{osp/⊘}\)) is then defined by

\[
i_{osp/⊘} = 0.
\]

Thus,

\[
i_{osp/⊘} = 1 - \frac{u}{\beta}(\lambda - 1).
\]
This case is valid only when $0 < i_{p/osp} < i_{osp/\odot} < i_{\odot/os} < 1$ (boundary conditions). By definition, we have, $m_p = i_{p/osp}$, $m_{osp} = i_{osp/\odot} - i_{p/osp}$ and $m_{os} = 1 - i_{\odot/os}$. From this expression, we compute the expression of the profit $\pi = pi_{p/osp}$ and obtain from the FOC that $p^* = av/2a(\lambda - 1)$ and $m^*_p = av/(2a(\lambda - 1)(a + \alpha \lambda))$. We also deduce the number of OSP and OS users at equilibrium : $m_{osp}^* = \frac{a^2v^2}{2a(\lambda - 1)(a + \alpha \lambda)^2}$ and $m_{os}^* = u/\beta(1 - \lambda)$. From that, we can compute the profit of the firm and the surplus of all categories of users. The optimal profit of the firm is then 

$$\pi = \int_0^{i_{p/osp}} U_p(i)di = \frac{a^2v^2(3 - 2(\lambda - 1)\lambda)}{8\alpha(\lambda - 1)^2(a + \alpha \lambda)^2};$$

$$W_{osp} = \int_{i_{p/osp}}^{i_{osp/\odot}} U_{osp}(i)di = \frac{a^2v^2(a + \alpha \lambda)^2}{8\alpha(\lambda - 1)(a + \alpha \lambda)^2};$$

$$W_{os} = \int_{i_{osp/\odot}}^{1} U_{os}(i)di = \frac{u^2}{2\beta(1 - \lambda)};$$

$$W_{users} = W_p + W_{osp} + W_{os}$$

The P-OSP case

In that situation, users adopt either the P platform or the OSP platform. This case is close to the first situation (P-OSP-OS). Considering increasing values of $i$, users adopt the P first and then the OSP software. As previously, the user (noted $i_{p/osp}$) indifferent between P and OSP software is defined by $U_p(i_{p/osp}) = U_{OSP}(i_{p/osp})$ and is characterized by $i_{p/osp} = (am_{osp} - p)/(a \lambda)$. Since all remaining users adopt the OSP, the user indifferent between the OSP and the OS software (noted $i_{osp/os}$, characterized by $U_{osp}(i_{osp/os}) = U_{os}(i_{osp/os})$) and defined by $i_{osp/os} = (u - v + \beta(\lambda - 1))/(a + \beta(\lambda - 1))$ as previously) is located outside the unitary segment. Then, this case is valid only if the following condition hold : $0 < i_{p/osp} < 1 < i_{osp/os}$ (boundary conditions). Beside, we need to check that all users get a positive utility when adopting the OSP. A sufficient condition for that is $U_{osp}(1) > 0$ (meaning that the user located at position 1 that get the least utility when adopting the OSP also gets a positive utility).

The expression of the profit is the same as previously $\pi = pi_{p/osp}$ and obtain from the FOC that $p^* = a/2$ and $m^*_p = a/2(a + \alpha \lambda)$, we also deduce the number of OSP users at equilibrium : $m_{osp}^* = 1 - m^*_p = 1 - a/2(a + \alpha \lambda)$. From that, we can compute the profit of the firm and the surplus of all categories of users. The optimal profit of the firm is then $\pi^* = a^2/4(a + \alpha \lambda)$. Putting together the boundary conditions and the second-order condition, we obtain the following existing conditions for this equilibrium : $u + \alpha < v + \alpha \lambda$ and $v + \alpha \lambda > a$. In these conditions, we can deduce the expression of P users’ surplus $W_p$, of OSP users’ surplus $W_{osp}$ of OS users’ surplus $W_{os}$, the total surplus of all users $W_{users}$ (with $W_{users} = W_p + W_{osp} + W_{os}$):
\[ W_p = \int_{i_p}^{i_{osp}} U_p(i) di = \frac{a(4\alpha \lambda + a(4\nu + \alpha(2\lambda - 1)))}{8(a + \alpha \lambda)^2} \]

\[ W_{osp} = \int_{i_p}^{i_{osp}} U_{osp}(i) di 
= \frac{1}{8} \left( \frac{-4\beta(-2\alpha + (\alpha - 2\nu)\beta)}{(a + \beta)^2} + \frac{4(\alpha - \nu)(\nu + \alpha + 2\nu\beta)}{(a + \beta)^2(\lambda - 1)} + \frac{4\alpha\beta^2\lambda}{(a + \beta)^2} + \frac{a^2(a + \alpha)}{(a + \alpha \lambda)^2} - \frac{a(a + 4\nu)}{(a + \alpha \lambda)^2} \right) \]

\[ W_{os} = 0 \]

The existing conditions for equilibria of type P-OSP-OS, P-OSP-⊘-OS and P-OSP are all mutually exclusive. This means that starting with one parameter set, only one type of equilibrium can occur.

**Comparison of the different outcomes**

**Summary of equilibrium outcomes**

The following table summarizes the different equilibrium outcomes (See Table 1 at page 15):

\[ \]
<table>
<thead>
<tr>
<th>Case</th>
<th>( m_p^* )</th>
<th>( m_{os}^* )</th>
<th>( m_{osp}^* )</th>
<th>( p^* )</th>
<th>( \pi^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P - OS case</td>
<td>( m_p^* = (v + \beta - u)/2(\alpha + \beta) )</td>
<td>( m_{os}^* = (u - v + 2\alpha + \beta)/2(\alpha + \beta) )</td>
<td>( m_{osp}^* = 0 )</td>
<td>( p^* = (v + \beta - u)/2 )</td>
<td>( \pi^* = (v + \beta - u)^2/4(\alpha + \beta) )</td>
</tr>
<tr>
<td>P - ( \oslash ) - OS case</td>
<td>( m_p^* = v/2\alpha )</td>
<td>( m_{os}^* = u/\beta )</td>
<td>( m_{osp}^* = 0 )</td>
<td>( p^* = v/2 )</td>
<td>( \pi^* = v^2/4\alpha )</td>
</tr>
<tr>
<td>P - OSP case</td>
<td>( m_p^* = a/2(\alpha + \alpha \lambda) )</td>
<td>( m_{os}^* = 0 )</td>
<td>( m_{osp}^* = 1 - a/2(\alpha + \alpha \lambda) )</td>
<td>( p^* = a/2 )</td>
<td>( \pi^* = a^2/4(\alpha + \alpha \lambda) )</td>
</tr>
<tr>
<td>P - OSP - OS case</td>
<td>( m_p^* = a(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1)(a + \alpha \lambda) )</td>
<td>( m_{os}^* = (a + 2\alpha \lambda)(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1)(a + \alpha \lambda) )</td>
<td>( m_{osp}^* = a(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1) )</td>
<td>( p^* = a(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1) )</td>
<td>( \pi^* = a^2(v - u + \beta(1 - \lambda))^2/4(\alpha + \beta)^2(\lambda - 1)^2(a + \alpha \lambda) )</td>
</tr>
<tr>
<td>P - OSP - ( \oslash ) - OS case</td>
<td>( m_p^* = av/(2\alpha(\lambda - 1)(a + \alpha \lambda)) )</td>
<td>( m_{os}^* = u/\beta(1 - \lambda) )</td>
<td>( m_{osp}^* = a\sqrt{{v(\alpha + 2\alpha \lambda)}}\sqrt{\lambda(\lambda - 1)}\sqrt{(\alpha + \alpha \lambda)^2} )</td>
<td>( p^* = av/2\alpha(\lambda - 1) )</td>
<td>( \pi^* = a^2v^2/(4\alpha^2(\lambda - 1)^2(a + \alpha \lambda)) )</td>
</tr>
</tbody>
</table>

Table 1: List of possible outcomes
Outcome comparison

The following table summarizes the possible switch when introducing the OSP (by comparing the equilibrium conditions with and without OSP. Only one switch is impossible. Conform with intuition, when all users adopted one platform without OSP (case P-OS), the introduction of the OSP cannot lead to a situation where some users would not adopt a platform (case P-OSP-⊘-OS). Yet, some equilibria are mutually exclusive (remember that P-OSP-OS, P-OSP-⊘-OS and P-OSP are all mutually exclusive on the one hand and that P-OS and P-⊘-OS are mutually exclusive on the other hand), the introduction of DL cannot lead to multiple equilibria.

<table>
<thead>
<tr>
<th>To ...</th>
<th>Equilibria without OSP</th>
<th>From ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-OSP</td>
<td>Possible</td>
<td>P-OS</td>
</tr>
<tr>
<td>P-OSP-OS</td>
<td>Impossible</td>
<td>P-OS</td>
</tr>
<tr>
<td>P-OSP-Rien-OS</td>
<td>Possible (conditions sup)</td>
<td>P-OS</td>
</tr>
</tbody>
</table>

Figure 5: Potential switches induced by the introduction of DL.

To be done...